MATH-SHU 350

Recitation 3: Random Variables

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Exercise 1. *Find the counter examples such that:*

1. $X_n \xrightarrow{\mathbb{P}} X$, *but* X_n *does not converge to* X *almost surely.*
2. $X_n \xrightarrow{a.s.} X$, *but* $\mathbb{E}[X_n]$ *does not converge to* $\mathbb{E}[X]$.

Exercise 2. *If* $(X_n)_{n\geqslant 1}$ *is a sequence of random variables such that* $X_n \xrightarrow{\mathbb{P}} X$, *where* X *is finite a.s. and* ϕ *is a continuous function, show that* $\phi(X_n) \xrightarrow{\mathbb{P}} \phi(X)$.

Exercise 3. *For a random variable* X *taking its values in* N_+, *show that its expectation is*

$$\mathbb{E}[X] = \sum_{n=1}^{\infty} \mathbb{P}[X \geqslant n].$$

Exercise 4. *Let* $(\mathbb{R}, \mathcal{B}, \mu)$ *be a probability space and* $f \geqslant 0$ *integrable on* $\mathbb{R}$. *Show that, for every* $\varepsilon > 0$, *there exists compact set* K *such that*

$$\int_K f \, d\mu \geqslant \int_{\mathbb{R}} f \, d\mu - \varepsilon.$$

Exercise 5. *(Erdös-Rény graph) We denote by* G_n *a random graph of* n *vertices, and every two vertices are connected independently by an edge with probability* p.

1. *Let* E_n *be the number of edges in* G_n. *Calculate* $\mathbb{E}[E_n]$ *and* $\mathrm{Var}[E_n]$;
2. *Let* T_n *be the number of triangles in* G_n. *Calculate* $\mathbb{E}[G_n]$.